Lecture 13

Multi-Junction Transmission Lines, Duality Principle

13.1 Multi-Junction Transmission Lines

By concatenating sections of transmission lines of different characteristic impedances, a large variety of devices such as resonators, filters, radiators, and matching networks can be formed. We will start with a single junction transmission line first. A good reference for such problem is the book by Collin [83], but much of the treatment here is not found in any textbooks.

13.1.1 Single-Junction Transmission Lines

Consider two transmission line connected at a single junction as shown in Figure 13.1. For simplicity, we assume that the transmission line to the right is infinitely long so that there is no reflected wave. And that the two transmission lines have different characteristic impedances, Z_{01} and Z_{02}.

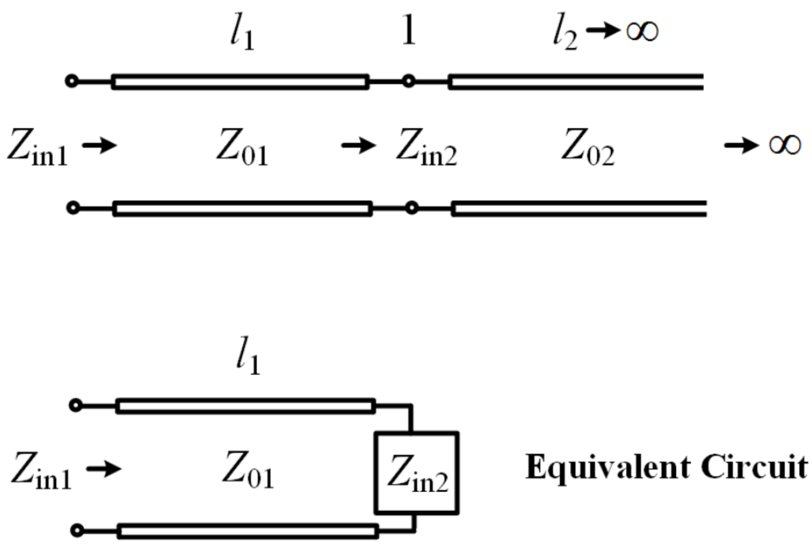

Figure 13.1: A single junction transmission line can be modeled by a equivalent transmission line terminated in a load Z_{in2}.

The impedance of the transmission line at junction 1 looking to the right,using the formula from previously derived,[1] is

$$Z_{in2} = Z_{02} \frac{1 + \Gamma_{L,\infty} e^{-2j\beta_2 l_2}}{1 - \Gamma_{L,\infty} e^{-2j\beta_2 l_2}} = Z_{02} \tag{13.1.1}$$

since no reflected wave exists, $\Gamma_{L,\infty} = 0$, the above is just Z_{02}. Transmission line 1 sees a load of $Z_L = Z_{in2} = Z_{02}$ hooked to its end. The equivalent circuit is shown in Figure 13.1 as well. Hence, we deduce that the reflection coefficient at junction 1 between line 1 and line 2, using the knowledge from the previous lecture, is Γ_{12}, and is given by

$$\Gamma_{12} = \frac{Z_L - Z_{01}}{Z_L + Z_{01}} = \frac{Z_{in2} - Z_{01}}{Z_{in2} + Z_{01}} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \tag{13.1.2}$$

13.1.2 Two-Junction Transmission Lines

Now, we look at the two-junction case. To this end, we first look at when line 2 is terminated by a load Z_L at its end as shown in Figure 13.2

[1]We should always remember that the relations between the reflection coefficient Γ and the normalized impedance Z_n are $\Gamma = \frac{Z_n - 1}{Z_n + 1}$ and $Z_n = \frac{1+\Gamma}{1-\Gamma}$.

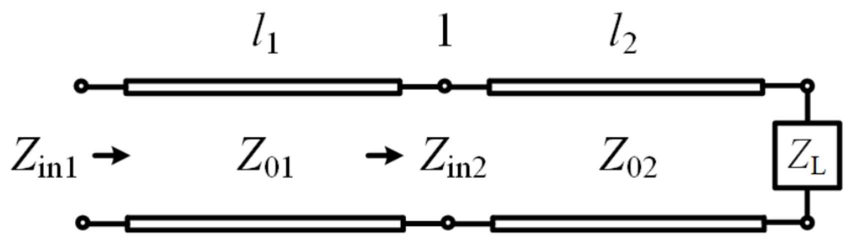

Figure 13.2: A single-junction transmission line with a load Z_L at the far end of the second line.

Then, using the formula derived in the previous lecture,

$$Z_{in2} = Z_{02} \frac{1 + \Gamma(-l_2)}{1 - \Gamma(-l_2)} = Z_{02} \frac{1 + \Gamma_{L2} e^{-2j\beta_2 l_2}}{1 - \Gamma_{L2} e^{-2j\beta_2 l_2}} \tag{13.1.3}$$

where we have used the fact that $\Gamma(-l_2) = \Gamma_{L2} e^{-2j\beta_2 l_2}$. It is to be noted that here, using knowledge from the previous lecture, that

$$\Gamma_{L2} = \frac{Z_L - Z_{02}}{Z_L + Z_{02}} \tag{13.1.4}$$

Now, line 1 sees a load of Z_{in2} hooked at its end. The equivalent circuit is the same as that shown in Figure 13.1. The generalized reflection coefficient at junction 1, which includes all the reflection of waves from its right, is now

$$\tilde{\Gamma}_{12} = \frac{Z_{in2} - Z_{01}}{Z_{in2} + Z_{01}} \tag{13.1.5}$$

Substituting (13.1.3) into (13.1.5), we have

$$\tilde{\Gamma}_{12} = \frac{Z_{02}(\frac{1+\Gamma}{1-\Gamma}) - Z_{01}}{Z_{02}(\frac{1+\Gamma}{1-\Gamma}) + Z_{01}} \tag{13.1.6}$$

where $\Gamma = \Gamma_{L2} e^{-2j\beta_2 l_2}$. The above can be rearranged to give

$$\tilde{\Gamma}_{12} = \frac{Z_{02}(1 + \Gamma) - Z_{01}(1 - \Gamma)}{Z_{02}(1 + \Gamma) + Z_{01}(1 - \Gamma)} \tag{13.1.7}$$

Finally, by further rearranging terms, it can be shown that the above becomes

$$\tilde{\Gamma}_{12} = \frac{\Gamma_{12} + \Gamma}{1 + \Gamma_{12}\Gamma} = \frac{\Gamma_{12} + \Gamma_{L2} e^{-2j\beta_2 l_2}}{1 + \Gamma_{12}\Gamma_{L2} e^{-2j\beta_2 l_2}} \tag{13.1.8}$$

where Γ_{12}, the local reflection coefficient, is given by (13.1.2), and $\Gamma = \Gamma_{L2}e^{-2j\beta_2 l_2}$ is the general reflection coefficient[2] at $z = -l_2$ due to the load Z_L. In other words,

$$\Gamma_{L2} = \frac{Z_L - Z_{02}}{Z_L + Z_{02}} \tag{13.1.9}$$

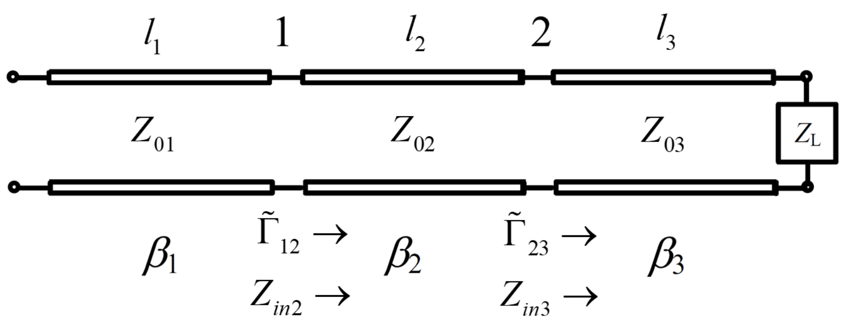

Figure 13.3: A two-junction transmission line with a load Z_L at the far end. The input impedance looking in from the far left can be found recursively.

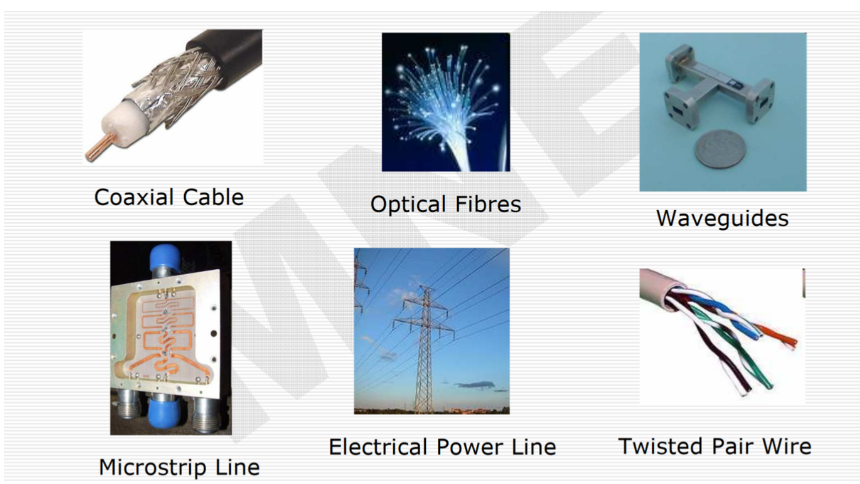

Figure 13.4: Different kinds of waveguides operating in different frequencies in power lines, RF, microwave, and optics. (courtesy of Owen Casha.)

[2]We will use the term "general reflection coefficient" to mean the ratio between the amplitudes of the left-traveling wave and the right-traveling wave on a transmission line.

Equation (13.1.8) is a powerful formula for multi-junction transmission lines. Imagine now that we add another section of transmission line as shown in Figure 13.3. We can use the aforementioned method to first find $\tilde{\Gamma}_{23}$, the generalized reflection coefficient at junction 2. Using formula (13.1.8), it is given by

$$\tilde{\Gamma}_{23} = \frac{\Gamma_{23} + \Gamma_{L3} e^{-2j\beta_3 l_3}}{1 + \Gamma_{23}\Gamma_{L3} e^{-2j\beta_3 l_3}} \tag{13.1.10}$$

where Γ_{L3} is the load reflection coefficient due to the load Z_L hooked to the end of transmission line 3 as shown in Figure 13.3. Here, it is given as

$$\Gamma_{L3} = \frac{Z_L - Z_{03}}{Z_L + Z_{03}} \tag{13.1.11}$$

Given the knowledge of $\tilde{\Gamma}_{23}$, we can use (13.1.8) again to find the new $\tilde{\Gamma}_{12}$ at junction 1. It is now

$$\tilde{\Gamma}_{12} = \frac{\Gamma_{12} + \tilde{\Gamma}_{23} e^{-2j\beta_2 l_2}}{1 + \Gamma_{12}\tilde{\Gamma}_{23} e^{-2j\beta_2 l_2}} \tag{13.1.12}$$

The equivalent circuit is again that shown in Figure 13.1. Therefore, we can use (13.1.8) recursively to find the generalized reflection coefficient for a multi-junction transmission line. Once the reflection coefficient is known, the impedance at that location can also be found. For instance, at junction 1, the impedance is now given by

$$Z_{in2} = Z_{01} \frac{1 + \tilde{\Gamma}_{12}}{1 - \tilde{\Gamma}_{12}} \tag{13.1.13}$$

instead of (13.1.3). In the above, Z_{01} is used because the generalized reflection coefficient $\tilde{\Gamma}_{12}$ is the total reflection coefficient for an incident wave from transmission line 1 that is sent toward the junction 1. Previously, Z_{02} was used in (13.1.3) because the reflection coefficients in that equation was for an incident wave sent from transmission line 2.

If the incident wave were to have come from line 2, then one can write Z_{in2} as

$$Z_{in2} = Z_{02} \frac{1 + \tilde{\Gamma}_{23} e^{-2j\beta_2 l_2}}{1 - \tilde{\Gamma}_{23} e^{-2j\beta_2 l_2}} \tag{13.1.14}$$

With some algebraic manipulation, it can be shown that (13.1.13) are (13.1.14) identical. But (13.1.13) is closer to an experimental scenario where one measures the reflection coefficient by sending a wave from line 1 with no knowledge of what is to the right of junction 1.

Transmission lines can be made easily in microwave integrated circuit (MIC) by etching or milling. A picture of a microstrip line waveguide or transmission line is shown in Figure 13.5.

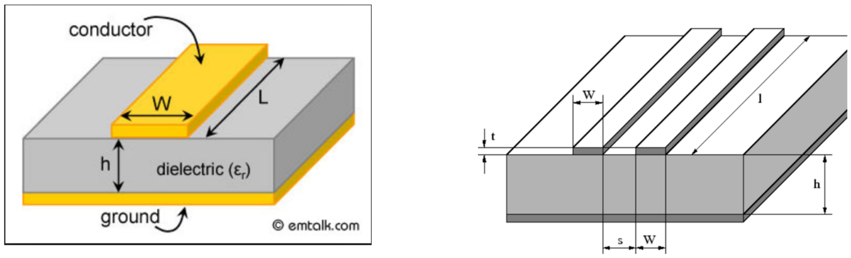

Figure 13.5: Schematic of a microstrip line with the signal line above, and a ground plane below (left). A strip line with each strip carrying currents of opposite polarity (right). A ground plane is not needed in this case.

13.1.3 Stray Capacitance and Inductance

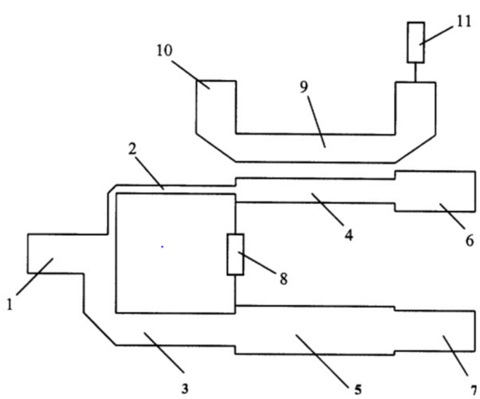

Figure 13.6: A general microwave integrated circuit with different kinds of elements.

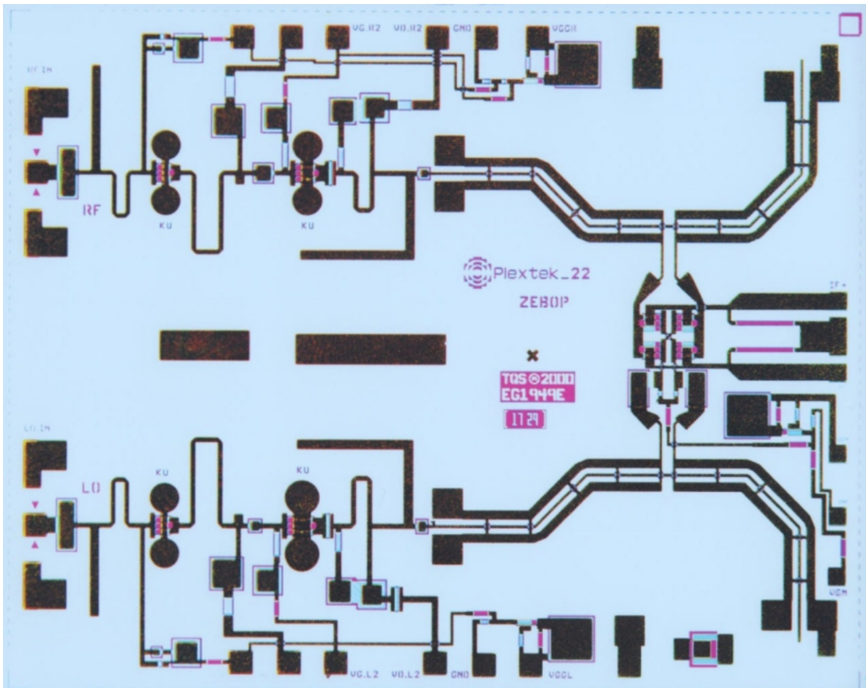

Figure 13.7: A generic microwave integrated circuit.

The junction between two transmission lines is not as simple as we have assumed. In the real world, or in MIC, the waveguide junction has discontinuities in line width, or shape. This can give rise to excess charge cumulation. Excess charge gives rise to excess electric field which corresponds to excess electric stored energy. This can be modeled by stray or parasitic capacitances.

Alternatively, there could be excess current flow that give rise to excess magnetic field. Excess magnetic field gives rise to excess magnetic stored energy. This can be modeled by stray or parasitic inductances. Hence, a junction can be approximated by a circuit model as shown in Figure 13.8 to account for these effects. The Smith chart or the method we have outlined above can still be used to solve for the input impedances of a transmission circuit when these parasitic circuit elements are added.

Notice that when the frequency is zero or low, these stray capacitances and inductances are negligible. But they are instrumental in modeling high frequency circuits.

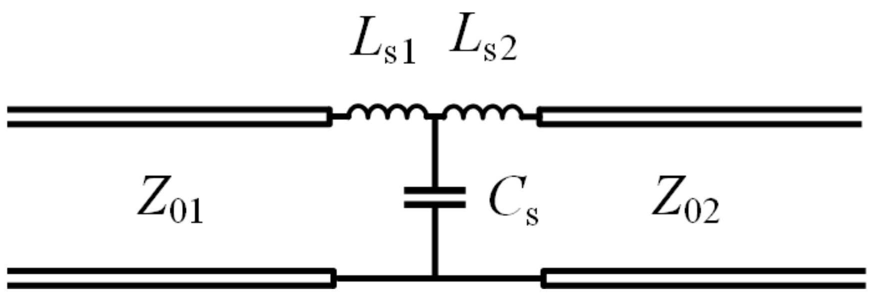

Figure 13.8: A junction between two microstrip lines can be modeled with a stray junction capacitance and stray inductances. The capacitance is used to account for excess charges at the junction, while the inductances model the excess current at the junction.

13.2 Duality Principle

Duality principle exploits the inherent symmetry of Maxwell's equations. Once a set of $\mathbf{E}$ and $\mathbf{H}$ has been found to solve Maxwell's equations for a certain geometry, another set for a similar geometry can be found by invoking this principle. Maxwell's equations in the frequency domain, including the fictitious magnetic sources, are

$$\nabla \times \mathbf{E}(\mathbf{r}, \omega) = -j\omega \mathbf{B}(\mathbf{r}, \omega) - \mathbf{M}(\mathbf{r}, \omega) \tag{13.2.1}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, \omega) = j\omega \mathbf{D}(\mathbf{r}, \omega) + \mathbf{J}(\mathbf{r}, \omega) \tag{13.2.2}$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}, \omega) = \varrho_m(\mathbf{r}, \omega) \tag{13.2.3}$$

$$\nabla \cdot \mathbf{D}(\mathbf{r}, \omega) = \varrho(\mathbf{r}, \omega) \tag{13.2.4}$$

One way to make Maxwell's equations invariant is to do the following substitution.

$$\mathbf{E} \to \mathbf{H}, \quad \mathbf{H} \to -\mathbf{E}, \quad \mathbf{D} \to \mathbf{B}, \quad \mathbf{B} \to -\mathbf{D} \tag{13.2.5}$$

$$\mathbf{M} \to -\mathbf{J}, \quad \mathbf{J} \to \mathbf{M}, \quad \varrho_m \to \varrho, \quad \varrho \to \varrho_m \tag{13.2.6}$$

The above swaps retain the right-hand rule for plane waves. When material media is included, such that $\mathbf{D} = \overline{\varepsilon} \cdot \mathbf{E}$, $\mathbf{B} = \overline{\boldsymbol{\mu}} \cdot \mathbf{H}$, for anisotropic media, Maxwell's equations become

$$\nabla \times \mathbf{E} = -j\omega \overline{\boldsymbol{\mu}} \cdot \mathbf{H} - \mathbf{M} \tag{13.2.7}$$

$$\nabla \times \mathbf{H} = j\omega \overline{\varepsilon} \cdot \mathbf{E} + \mathbf{J} \tag{13.2.8}$$

$$\nabla \cdot \overline{\boldsymbol{\mu}} \cdot \mathbf{H} = \varrho_m \tag{13.2.9}$$

$$\nabla \cdot \overline{\varepsilon} \cdot \mathbf{E} = \varrho \tag{13.2.10}$$

In addition to the above swaps, one need further to swap for material parameters, namely,

$$\overline{\boldsymbol{\mu}} \to \overline{\varepsilon}, \quad \overline{\varepsilon} \to \overline{\boldsymbol{\mu}} \tag{13.2.11}$$

13.2.1 Unusual Swaps

If one adopts swaps where seemingly the right-hand rule is not preserved, e.g.,

$$\mathbf{E} \rightarrow \mathbf{H},\ \mathbf{H} \rightarrow \mathbf{E},\ \mathbf{M} \rightarrow -\mathbf{J},\ \mathbf{J} \rightarrow -\mathbf{M}, \tag{13.2.12}$$

$$\varrho_m \rightarrow -\varrho,\ \varrho \rightarrow -\varrho_m,\ \overline{\boldsymbol{\mu}} \rightarrow -\overline{\boldsymbol{\varepsilon}},\ \overline{\boldsymbol{\varepsilon}} \rightarrow -\overline{\boldsymbol{\mu}} \tag{13.2.13}$$

The above swaps will leave Maxwell's equations invariant, but when applied to a plane wave, the right-hand rule seems violated.

The deeper reason is that solutions to Maxwell's equations are not unique, since there is a time-forward as well as a time-reverse solution. In the frequency domain, this shows up in the choice of the sign of the $\mathbf{k}$ vector where in a plane wave $k = \pm\omega\sqrt{\mu\varepsilon}$. When one does a swap of $\mu \rightarrow -\varepsilon$ and $\varepsilon \rightarrow -\mu$, k is still indeterminate, and one can always choose a root where the right-hand rule is retained.

13.2.2 Fictitious Magnetic Currents

Even though magnetic charges or monopoles do not exist, magnetic dipoles do. For instance, a magnet can be regarded as a magnetic dipole. Also, it is believed that electrons have spins, and these spins make electrons behave like tiny magnetic dipoles in the presence of a magnetic field.

Also if we form electric current into a loop, it produces a magnetic field that looks like the electric field of an electric dipole. This resembles a magnetic dipole field. Hence, a magnetic dipole can be made using a small electric current loop (see Figure 13.9).

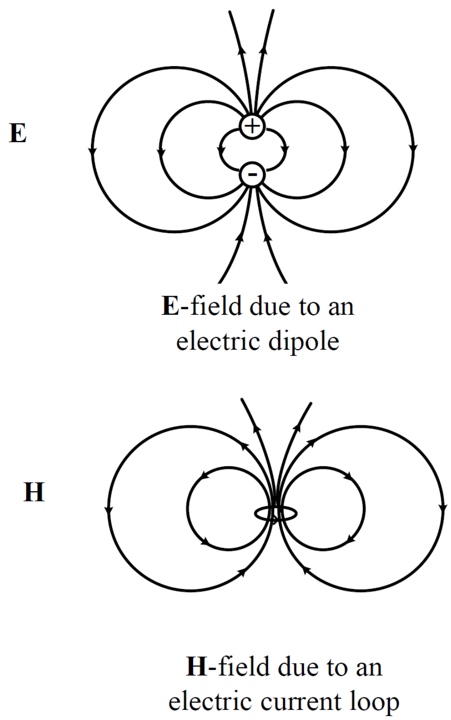

Figure 13.9: Sketches of the electric field due to an electric dipole and the magnetic field due to a electric current loop. The **E** and **H** fields have the same pattern, and can be described by the same formula.

Because of these similarities, it is common to introduce fictitious magnetic charges and magnetic currents into Maxwell's equations. One can think that these magnetic charges always occur in pair and together. Thus, they do not contradict the absence of magnetic monopole.

The electric current loops can be connected in series to make a toroidal antenna as shown in Figure 13.10. The toroidal antenna is used to drive a current in an electric dipole. Notice that the toroidal antenna acts as the primary winding of a transformer circuit.

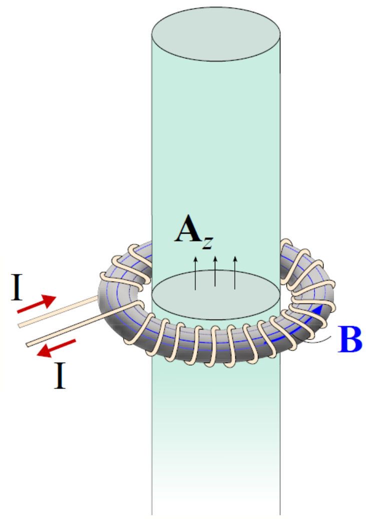

Figure 13.10: A toroidal antenna used to drive an electric current through a conducting cylinder of a dipole. One can think of them as the primary and secondary turns of a transformer (courtesy of Q.S. Liu).

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1820, 10 juin 1822, 22 décembre 1823, 12 septembre et 21 novembre 1825. Bachelier, 1825.

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